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Transient Optical Regime of Two Level Atom in Laser Light

Smail Bougouffa[1,2] and Saud Al-Awfi[1]

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An extended analytic approach is considered for optical Bloch equations in the two level atom interacting with laser light. The separation approach of coupled differential equations is always possible with a sequence of special transformation into the Riccati nonlinear differential equation. The conditions that permit an exact solutions of three coupled system are extracted in a natural manner. The case of sodium atom moving along the axis of a monochromatic wave is treated in some details including a discussion on the radiation pressure forces exerted by laser light in the transient regime.

KEY WORDS: optical Bloch equations; light-atom interactions; coupled differential equations; transient regime; mechanical effects of light; dissipative and reactive forces.

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1. INTRODUCTION

It is well known that the study of the mechanical effects of light on two- and three- level atoms has several important goals: it leads to a simple categorization of the forces acting on the atomic center of mass, and in addition permits a simple discussion of cooling and trapping.

In order to investigate concretely the limits of the recent expanding research field, called laser cooling and trapping, several theoretical treatments have been proposed. The review of some of these approaches and the comparison of their advantages, difficulties and domains of validity, can be found in the literatures (Cohen-Tannoudji, 1992; Gordon and Ashkin, 1980; Meystre and Sargent, 1999).

In general the physical mechanisms of the atomic motion in light are governed by the optical Bloch equations and they are solved in the steady-state, to

[1] Department of physics, Faculty of Science, P.O. Box 344, Taibah University, Madinah, Saudi Arabia.
[2] To whom correspondence should be addressed at Department of physics, Faculty of Science, P.O. Box 344, Taibah University, Madinah, Saudi Arabia; e-mail:sbougouffa@hotmail.com.

illustrate the mean radiative forces for a two- and three-level atom. In this case the range of the validity of these solutions is assumed to be in the same order of the characteristic time required by the internal state of the atom to reach its steady state.

On the other hand, to show the transient optical effects on atomic motion, we have to solve the OBE which form a set of coupled linear first-order differential equations. This coupled system does not, in general, permit exact analytic solutions and one must employ some approximations to reduce this system to a tractable form. Indeed, the problem of the two-level system leads to a three coupled differential equations with constant coefficients which can in principle be solved exactly. However, the characteristic equation of this system is a cubic one and its exact analytic roots do not lend themselves to analytic treatment, and some approximations or graphical representations must be employed to reduce the roots into a more manageable form in order to obtain analytic solutions (Carter *et al*., 2005). On the other hand, the Bloch equations with variable coefficients can not analytically be solved (Cohen-Tannoudji, 1992). It may, however, be worthwhile if the physical models can be constructed in such a manner that the coupled system can either be analytically solved or transformed into another system in which the equations are decoupled and solvable separately.

In this work, the emphasis is laid to determine the conditions that permit exact analytic solutions for the optical Bloch equations with a two-level atom, using the nonlinear Riccati equation. We shall present a new treatment of solution with reference to a three coupled Bloch equations which is important in many applications and exhibited interesting features including transient optical angular momentum effects (Carter *et al*., 2005), transient response of cold atomic beam (Livesey *et al*., 2003), atomic cooling schemes such as sub-Doppler cooling (Xu *et al*., 2003), laser cooling of atom, ions or molecules (Vuletic and Chu, 2000; Robins *et al*., 2001) and ultra-cold atomic samples, as can be now obtained by several cooling methods (Wineland and Itano, 1987). Within this scope, this paper begins in Section 2 with setting up the mathematical structure of the two-level atom with stable ground state and derives the optical Bloch equations (OBE). Section 3 deals with the suggested approach where the OBE are converted to a system of coupled nonlinear Riccati equations. We derive the conditions that permit the total separation of these equations and the closed analytic solutions are obtained for the Bloch vector. In Section 4, an example is considered to illustrate the usefulness of this treatment. The atomic motion in a plane running wave will be pointed out. The essential features of the forces will be reported for the Na atom, and in Section 5 we conclude with some remarks and discussion, and possible extension of the area of investigations.

2. TWO-LEVEL ATOM WITH STABLE GROUNDSTATE

This section is devoted to further analysis of two-level atoms. The Hamiltonian of the global system can be written (Cohen-Tannoudji, 1992; Meystre and Sargent, 1999) as

$$H = H_A + H_V + H_{AL} + H_{AV} \tag{1}$$

where H_A is the atomic Hamiltonian, which is the sum of the center-of-mass kinetic energy of the atom and the internal transition energy. The second term is the energy of the quantum radiation field initially in the vacuum state. The third term is the coupling of the atomic dipole $\mathbf{d}$ and the laser electric field $\mathbf{E}_l(\mathbf{R}, t)$. Finally, the last term describes the atom-vacuum coupling.

In the semiclassical approximation and with the rotating wave approximation, for the pure radiative decay which is defined by an upper-to-lower-level transition with the dipole constant $\gamma = \frac{1}{2}\Gamma$ as in (Fig. 1), the time evolution of the density operator is given by

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] - \frac{\Gamma}{2}[\sigma_+\sigma_-\rho + \rho\sigma_+\sigma -] + \Gamma\sigma_-\rho\sigma_+ \tag{2}$$

where $\sigma_\pm$ are the Pauli spin-flip operators. By choosing the energy zero to be half between the upper and lower levels and projecting onto the bare state $|\, a\rangle, |\, b\rangle$

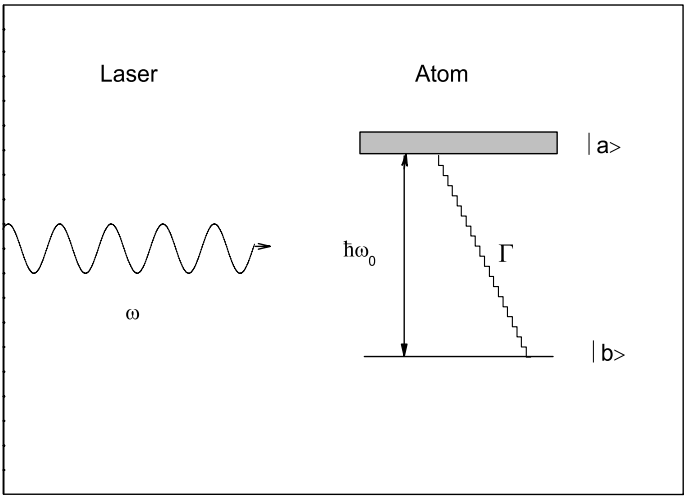

Fig. 1. A two-level atom with a stable ground state and upper level decay rate Γ. The atom of energy $\hbar\omega_0$ is assumed to interact with laser light of frequency ω.

basis, the density matrix equations can be obtained. By introducing the Bloch vector (U, V, W), it is easy to show (Cohen-Tannoudji, 1992; Meystre and Sargent, 1999) that the components of the Bloch vector for upper to lower decay satisfy the following motion equations

$$\begin{pmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{pmatrix} = \begin{pmatrix} -\Gamma/2 & (\delta + \dot{\phi}(\mathbf{r})) & 0 \\ -(\delta + \dot{\phi}(\mathbf{r})) & -\Gamma/2 & -\mathcal{R}_0(\mathbf{r}) \\ 0 & \mathcal{R}_0(\mathbf{r}) & -\Gamma \end{pmatrix} \begin{pmatrix} U \\ V \\ W \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\Gamma \end{pmatrix} \tag{3}$$

where the frequency shift $\delta = \omega - \omega_0$ is the detuning, ϕ is the phase of the electric field $\mathbf{E}(\mathbf{r}, t)$, where we lump all rapid spatial dependence of the field into the phase $\phi(\mathbf{r})$ and the Rabi frequency $\mathcal{R}_0(\mathbf{r})$ is now a function of position in general. In terms of the components of the Bloch vector the mean radiative force for a two-level atom is simply given by

$$\mathcal{F}(\mathbf{r}, t) = -\frac{1}{2}\hbar\mathcal{R}_0(\mathbf{r})[U(\mathbf{r}, t)\alpha(\mathbf{r}, t) + V(\mathbf{r}, t)\beta(\mathbf{r}, t)] \tag{4}$$

where we have introduced

$$\alpha = \frac{\nabla\mathcal{R}_0(\mathbf{r})}{\mathcal{R}_0(\mathbf{r})} \tag{5}$$

and

$$\beta = \nabla\phi(\mathbf{r}) \tag{6}$$

On the other hand, the first component of the Bloch vector, U, is proportional to the real part of ρ_{ab} and is responsible for dispersive effects (index of refraction.) In contrast, V is proportional to the imaginary part of ρ_{ab} and hence is responsible for amplification and absorption of light. This distinction permits us to separate the $\mathcal{F}(\mathbf{r}, t)$ into a dissipative and dispersive contribution,

$$\mathcal{F}(\mathbf{r}, t) = \mathcal{F}_{\text{dissip}}(\mathbf{r}, t) + \mathcal{F}_{\text{react}}(\mathbf{r}, t) \tag{7}$$

where we have introduced the dissipative, or radiative pressure force

$$\mathcal{F}_{\text{dissip}} = -\frac{1}{2}\hbar\mathcal{R}_0 V\beta \tag{8}$$

and the reactive, or dipole force

$$\mathcal{F}_{\text{react}} = -\frac{1}{2}\hbar\mathcal{R}_0 U\alpha \tag{9}$$

From the definitions of α and β we note that the dissipation force is non-vanishing only if the laser field exhibits a phase gradient, $\beta \neq 0$. In contrast, the reactive force requires the presence of a field gradient, $\alpha \neq 0$. The steady-state solution to the equations (3) can be obtained by putting

$$\dot{U} = \dot{V} = \dot{W} = \dot{\phi} = 0 \tag{10}$$

and solving the linear system,

$$\begin{pmatrix} -\Gamma/2 & \delta & 0 \\ -\delta & -\Gamma/2 & -\mathcal{R}_0 \\ 0 & \mathcal{R}_0 & -\Gamma \end{pmatrix} \begin{pmatrix} U_{st} \\ V_{st} \\ W_{st} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\Gamma \end{pmatrix} \tag{11}$$

In general, from these solutions the reactive force can be deduced (Cohen-Tannoudji, 1992; Meystre and Sargent, 1999)

$$\mathcal{F}_{\text{react}} = -\frac{\hbar\delta}{4}\left(\frac{\nabla\mathcal{R}_0^2}{\delta^2 + (\Gamma/2)^2 + \mathcal{R}_0^2/2}\right) \tag{12}$$

which shows that the reactive force is proportional to the sign of the detuning between the atomic transition frequency and the light frequency $\delta = \omega - \omega_0$. Note finally that the $\mathcal{F}_{\text{react}}$ derives from an optical potential U_{opt} that can be given by

$$U_{\text{opt}} = \frac{\hbar\delta}{2}\ln\left(1 + \frac{\mathcal{R}_0^2/2}{\delta^2 + (\Gamma/2)^2}\right) \tag{13}$$

since one can write

$$\mathcal{F}_{\text{react}} = -\nabla U_{\text{opt}} \tag{14}$$

For $\delta < 0$, which is called red detuning in that the atom becomes strong-field seeking, that is, $\mathcal{F}_{\text{react}}$ forces the atom towards regions of higher laser intensity. For blue detuning $\delta > 0$, in contrast, the force is repulsive and forces the atoms towards regions of weak laser intensity. Note also that $\mathcal{F}_{\text{react}}$ does not saturate for increasing Rabi frequencies.

In the following section, we will try to present a new treatment of the coupled differential Eq. (3) to investigate the transient effects of the dissipative and dipole forces.

3. SEPARATION OF THE OPTICAL BLOCH EQUATIONS

The system (3) can be written in the matrix form as

$$\dot{\mathbf{B}} = \mathbf{M}\mathbf{B} + \mathbf{C} \tag{15}$$

where $\mathbf{B}$ is the Bloch vector, $\mathbf{M}$ is a non-diagonal 3×3 matrix and $\mathbf{C}$ is inhomogeneous vector. The general solution of the system (15) is given by

$$\mathbf{B} = \mathbf{B}_h + \mathbf{B}_p \tag{16}$$

where $\mathbf{B}_h$ is the homogenous solution of the system without the term $\mathbf{C}$ and $\mathbf{B}_p$ is the particular solution. By choosing $\dot{\mathbf{B}} = 0$ in the system (15), the particular

solution can be obtained $\mathbf{B}_p = -\mathbf{M}^{-1}\mathbf{C}$ which is the steady-state solution,

$$\mathbf{B}_p = \begin{pmatrix} U_{st} \\ V_{st} \\ W_{st} \end{pmatrix} = \begin{pmatrix} \frac{2\delta}{\mathcal{R}_0(\mathbf{r})}\frac{s}{1+s} \\ \frac{\Gamma}{\mathcal{R}_0(\mathbf{r})}\frac{s}{1+s} \\ -\frac{1}{1+s} \end{pmatrix} \tag{17}$$

where s is a saturation parameter

$$s = \frac{1}{2}\frac{\mathcal{R}_0(\mathbf{r})}{(\Gamma/2)^2 + \delta^2} \tag{18}$$

Consider now the homogeneous system

$$\dot{\mathbf{B}}_h = \mathbf{M}\mathbf{B}_h \tag{19}$$

and for simplification we shall introduce the following notations $a = -\Gamma/2$, $b = \delta + \dot{\phi}$, and $c = -\mathcal{R}_0$. Then the matrix $\mathbf{M}$ can be written as

$$\mathbf{M} = \begin{pmatrix} a & b & 0 \\ -b & a & c \\ 0 & -c & 2a \end{pmatrix} \tag{20}$$

In general the separation operation is a transformation $\mathcal{T}$ such that the original basis $\mathcal{B}$ is transformed into a new one, $\mathcal{X} = \mathcal{T}\mathcal{B}$, which gives in principle another system of differential equations that can be separated. The separation operation is, however, not always possible unless some constraints on the elements of $\mathbf{M}$ are imposed, and the purpose of the method proposed here is to find such constraints and thence the solutions of the homogeneous system. This treatment amounts to transforming the homogeneous system into a set of nonlinear Riccati equations (Bougouffa and Kamli, 2004) that can be delighted without difficulty. We shall take trial solutions of the form

$$U_h = X_1(t)V_h \tag{21}$$

$$W_h = X_2(t)V_h \tag{22}$$

where X's are any continuous functions of time to be determined. However, for notational convenience we will drop the index h and restore it at the end of calculations. Substituting from (21) and (22) in Eq. (19) we get the following three equations for the component V:

$$\dot{V} = \left(a + \frac{b}{X_1} - \frac{\dot{X}_1}{X_1}\right) V \tag{23}$$

$$\dot{V} = (a + cX_2 - bX_1)\, V \tag{24}$$

$$\dot{V} = \left(2a - \frac{c}{X_2} - \frac{\dot{X}_2}{X_2}\right) V \tag{25}$$

These three equations must be identical since they govern the same component V. The compatibility of these equations leads to coupled nonlinear differential equations for the functions X_1 and X_2. By identifying (23) with (24) and (25) with (24) we get for X_1 and X_2 the equations

$$-\dot{X}_1 + bX_1^2 = cX_1X_2 - b \tag{26}$$

$$\dot{X}_2 - aX_2 + cX_2^2 = bX_1X_2 - c \tag{27}$$

Equations (26) and (27) are of the Riccati equations form except for the coupled term involving the product of the two functions; X_1X_2. It is clear that this system as it stands is not separable. To separate these equations we shall seek solutions of the form

$$X_1X_2 = \varphi(t) \tag{28}$$

where $\varphi(t)$ is a function to be determined. With this assumption a general transformation for canceling $\varphi(t)$ is not yet known (Bougouffa and Kamli, 2004). Consequently, the simplest nontrivial situation which allows the separation of Eqs. (26) and (27)is that the function $\varphi(t)$ equals a constant α, i.e. $X_1X_2 = \alpha$. With this assumption Eqs. (26) and (27) become

$$-\dot{X}_1 + bX_1^2 = c\alpha - b \tag{29}$$

$$\dot{X}_2 - aX_2 + cX_2^2 = b\alpha - c \tag{30}$$

The Riccati equations (28) and (29) can now be solved using the substitutions

$$X_1 = -\frac{1}{b}\frac{\dot{Z}_1}{Z_1} \tag{31}$$

$$X_2 = \frac{1}{c}\frac{\dot{Z}_2}{Z_2} \tag{32}$$

The functions Z_1 and Z_2 satisfy the following second-order equations:

$$\ddot{Z}_1 + b(b - c\alpha)Z_1 = 0 \tag{33}$$

$$\ddot{Z}_2 - a\dot{Z}_2 + c(c - b\alpha)Z_2 = 0 \tag{34}$$

These two equations are readily solved and give

$$Z_1 = A_+e^{+i\sqrt{b(b-c\alpha)}t} + A-e^{-i\sqrt{b(b-c\alpha)}t} \tag{35}$$

$$Z_2 = e^{\frac{a}{2}t}(B_+e^{+i\mu t} + B_-e^{-i\mu t}) \tag{36}$$

where A_+, A_-, B_+, B_- are the constants and

$$\mu = \sqrt{4c(c - b\alpha) - a^2} \tag{37}$$

Now using Eqs. (24, 30–35) we can solve for the components of the Bloch vector as

$$V = V_0 Z_1 Z_2 e^{at} \tag{38}$$

with $V(t = 0) = V_0$. From Eqs. (21, 22) we obtain

$$U = -\frac{V_0}{b} \dot{Z}_1 Z_2 e^{at} \tag{39}$$

$$W = \frac{V_0}{c} \dot{Z}_2 Z_1 e^{at} \tag{40}$$

The value of the constant α is determined as follows, since Eqs. (23) and (25) are identical, the terms inside the square brackets must be equal. Equating these two brackets and using (28) we obtain the following equation for X_1:

$$-\dot{X}_1 + \frac{c}{2\alpha} X_1^2 = \frac{a}{2} X_1 - \frac{b}{2} \tag{41}$$

Comparing Eqs. (29) and (41) we obtain the first condition by identification:

$$a = 0 \tag{42}$$

which corresponds to the case with no decay. In this case the solutions of the coupled system can be used in the discussions of optical nutations, free induction decay, photon echo, and Ramsey fringes (Meystre and Sargent, 1999).

On the other hand, it can be seen that if X_1 is a solution of the Eq. (29), then βX_1 satisfies the following equation

$$-\dot{X}_1 + b\beta X_1^2 = (c\alpha - b)/\beta \tag{43}$$

where β is a constant. By identification term to term between this equation and (41) we obtain the following conditions,

$$b\beta = \frac{c}{2\alpha} \tag{44}$$

$$\frac{c\alpha - b}{\beta} = -\frac{b}{2} \tag{45}$$

Solving these equations for α, we obtain

$$\alpha_{1,2} = \frac{1}{2} \left[\frac{b}{c} \pm \sqrt{\left(\frac{b}{c}\right)^2 - 1} \right] \tag{46}$$

It remains to determine the constants of integration A_+, A_-, B_+ and B_- we need four equations. These are provided from the initial conditions of the problem

$$U(t=0) = U_0 \tag{47}$$

$$V(t=0 = V_0 \tag{48}$$

$$W(t=0) = W_0 \tag{49}$$

and the condition $X_1 X_2 = \alpha$. After some simple algebra, we obtain the exact expressions of the Bloch component vector.

$$U(t) = \frac{(c^2 + b^2\cos(\Omega t))}{\Omega^2} U_0 + \frac{b}{\Omega}\sin(\Omega t) V_0 + \frac{bc(1-\cos(\Omega t))}{\Omega^2} W_0 \tag{50}$$

$$V(t) = -\frac{b}{\Omega}\sin(\Omega t) U_0 + \cos(\Omega t) V_0 + \frac{c}{\Omega}\sin(\Omega t) W_0 \tag{51}$$

$$W(t) = \frac{bc(1-\cos(\Omega t))}{\Omega^2} U_0 - \frac{c}{\Omega}\sin(\Omega t) V_0 + \frac{(b^2 + c^2\cos(\Omega t))}{\Omega^2} W_0 \tag{52}$$

where Ω is the generalized Rabi flopping frequency

$$\Omega = \sqrt{b^2 + c^2} = \sqrt{\delta^2 + \mathcal{R}_0^2} \tag{53}$$

Another constructive solution to the Bloch equations, particularly for inhomogeneous broadened problems similar to the free induction decay and photon echo, is the free progression with no applied field ($\mathcal{R}_0 = 0$, i.e. $c = 0$). In the limit $a = 0$, the solution of the Bloch equations can also be given by Eqs. (50–52) with $c = 0$, where the third component W becomes a constant of the motion. These steps comprise the analytic approach to solve the system of coupled differential equations.

Consider now the case where $\varphi(t)$ is not constant. For simplicity, we can rewrite the system (19) with the following notations,

$$U = \bar{U} e^{at}, \quad V = \bar{V} e^{at}, \quad W = \bar{W} e^{2at} \tag{54}$$

and we shall take trial solutions of the form

$$\bar{U} = X_1(t)\bar{V} \tag{55}$$

$$\bar{W} = X_2(t)\bar{V} \tag{56}$$

then we get the following three equations for the component $\bar{V}$:

$$\dot{\bar{V}} = \left(\frac{b - \dot{X}_1}{X_1}\right)\bar{V} \tag{57}$$

$$\dot{\bar{V}} = \left(-bX_1 + ce^{at}X_2\right)\bar{V} \tag{58}$$

$$\dot{\bar{V}} = -\left(\frac{ce^{-at} + \dot{X}_2}{X_2}\right)\bar{V} \tag{59}$$

As above, these three equations must be identical since they govern the same component $\bar{V}$. By identifying these equations we get for X_1 and X_2 three differential equations

$$\frac{b - \dot{X}_1}{X_1} = -bX_1 + ce^{at}X_2 \tag{60}$$

$$-\frac{ce^{-at} + \dot{X}_2}{X_2} = -bX_1 + ce^{at}X_2 \tag{61}$$

$$\frac{b - \dot{X}_1}{X_1} = -\frac{ce^{-at} + \dot{X}_2}{X_2} \tag{62}$$

In the last equation we assume the ansatz

$$\frac{\dot{X}_1}{X_1} - \frac{\dot{X}_2}{X_2} = \alpha \tag{63}$$

then

$$\frac{X_1}{X_2} = e^{\alpha t} \tag{64}$$

where α is a constant to be determined. With this new assumption the solutions of the Eq. (56) become

$$X_1 = \frac{1}{\alpha}\left[b + ce^{(\alpha - a)t}\right] \tag{65}$$

$$X_2 = \frac{e^{\alpha t}}{\alpha}\left[b + ce^{(\alpha - a)t}\right] \tag{66}$$

Substituting these solutions in the Eqs. (54) and (55) we find

$$[c\alpha(\alpha - a) - 2b^2c + c^3]e^{(\alpha-a)t} + cb^2e^{-(\alpha-a)t} - bc^2e^{2(\alpha-a)t} = b\alpha^2 + b^3 - 2bc^2 \tag{67}$$

To solve this equation we can choose $\alpha = a$, which implies the following condition on the parameters a, b, and c

$$a = (c + b)\sqrt{\frac{c - b}{b}} \tag{68}$$

Finally we can state that the system (19) can completely be separated without increasing its order of differentiation and solved if and only if the parameters a, b and c satisfy the condition (68). Indeed, with this condition the homogenous

solutions of the system (19) are

$$U = U_0 e^{at} e^{(\frac{ab}{b+c})t} \tag{69}$$

$$V = V_0 e^{at} e^{(\frac{ab}{b+c})t} \tag{70}$$

$$W = W_0 e^{at} e^{(\frac{ab}{b+c})t} \tag{71}$$

with $U_0 = W_0 = (\frac{b+c}{a})V_0$ and U_0, V_0, W_0 are the initial values of the Bloch vector components. Equations (69)–(71) are the desired solutions. They are exact solutions in the context of the conditions (68). The three coupled Bloch equations are also applicable to the three level atom interacting with cavity single mode field. A similar set of equations have also been derived in Garraway and Kinght (1996) for multi-mode field using the pseudo mode and coupling factorization where approximate solutions were obtained for the system of equations, in contrast the method suggested here gives exact analytic solutions under the same conditions. Since we know the Bloch vector components will be real, we choose the parameter a in terms of the others b and c from the condition (68)to be real, then the quantity should be positive $(\frac{c-b}{b}) > 0$,which is satisfied only for the red detuning.

4. MOVING ATOM IN LASER PLANE WAVE

To illustrate the efficiency of the above approach, we shall consider here an atom moving with a velocity $\mathbf{v}_0$ along the axis of a monochromatic wave with vector $\mathbf{k}$. Treating its center-of-mass classically, then the amplitude and the polarization of the electric field do not depend on $\mathbf{r}$. The Rabi frequency does not depend on time

$$\mathcal{R}_0(\mathbf{r} = \mathbf{v}_0 t) = \mathcal{R}_0 = constant \tag{72}$$

On the other hand, the phase ϕ varies linearly with $\mathbf{r}$

$$\phi = -\mathbf{k}.\mathbf{r} \tag{73}$$

so that

$$\dot{\phi} = -\frac{d\mathbf{r}}{dt}.\nabla\phi = \mathbf{v}_0.\nabla\phi = -\mathbf{k}.\mathbf{v}_0 \tag{74}$$

Since $\mathcal{R}_0$ and $\dot{\phi}$ are time independent, the optical Bloch equations are still a system of coupled linear differential equations with time independent coefficients. They thus have a general solutions which are derived from the solutions obtained in the precedent sections by the substitution

$$\delta \to \delta + \dot{\phi} = \delta - \mathbf{k}.\mathbf{v}_0 \tag{75}$$

or equivalently, since $\delta = \omega - \omega_0$ by the substitution

$$\omega \to \omega - \mathbf{k}.\mathbf{v}_0 \tag{76}$$

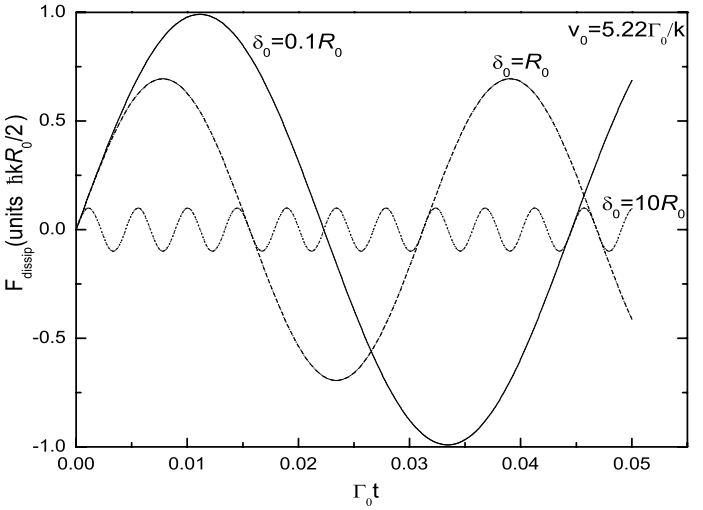

Fig. 2. The mean dissipative force in terms of $\Gamma_0 t$ with different values of detuning δ_0 for the velocity of atom $v_0 = 5.22\Gamma_0/k$. The initial conditions and parameters are given in the text.

such a result means that the atom moving with velocity $\mathbf{v}_0$ sees the laser frequency shifted by the Doppler $-\mathbf{k}.\mathbf{v}_0$.

The mean dissipative force experienced by the atom is evaluated numerically for the sodium atom (Al-Awfi and Babiker, 1996a,b) with the related parameters ($\Gamma_0 = 6.13X10^6 s^{-1}, k = 10.67X10^6 m^{-1}, \mathcal{R}_0 = 8.56X10^9 s^{-1}$) and the results are displayed in Figs. 2, 3, 4 and 5. Figure 2 exhibits the dynamics of the dissipative force in the case of no decay ($\Gamma = 0$) and with the initial conditions $U_0 = V_0 = 0$ and $W_0 = 1$, for different values of detuning δ_0 with the velocity of the atom $v_0 = 5.22\Gamma_0/k$. the dynamics depicts a regular oscillatory behavior and decreases whereas the detuning increases. We note that the transient regime becomes apparently permanent, while in this limit of no decay, the dissipative force is equal zero in the steady state. In Fig. 3. we present the variations of the dissipative force in terms of the velocity v_0 of the atom at fixed time ($\Gamma_0 t = 0.5$) for different values of detuning δ_0. The oscillatory behavior is also pronounced in the transient regime and the force exhibits declination when the detuning increases. We note that at this time for large values of velocity, the force starts to collapse. Figure 4. represents $\mathcal{F}_{\text{dissip}}$ in terms of the detuning δ at different times $\Gamma_0 t$. they show the decrease oscillatory behavior as δ increases in the red and blue detuning regions. Figure 5. shows the dissipative force as functions of time for different values of the detuning in the red detuning region which is the condition imposed by the equation (68). The behavior represents the decay to the stationary regime which is the steady

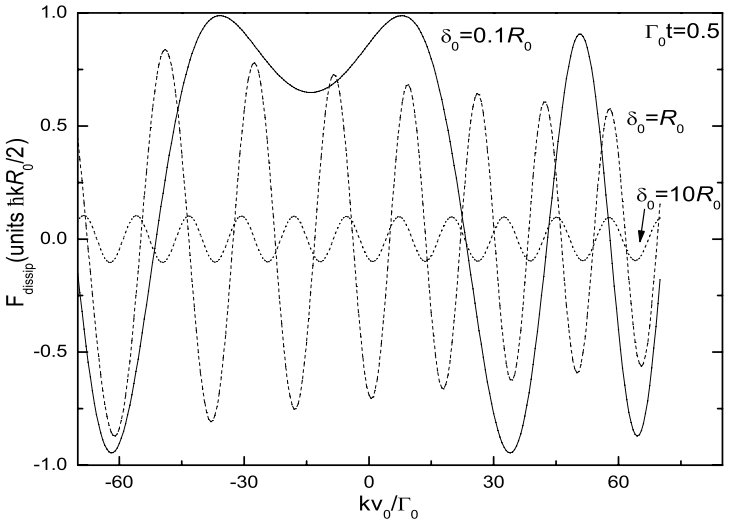

Fig. 3. The mean dissipative force in terms of kv_0/Γ_0 with different values of detuning δ_0 for $\Gamma_0 t = 0.5$. The initial conditions and parameters are given in the text.

state about a short time. The numerical calculation are done in this case with the initial condition $V(t = 0) = 0$. In contrast to the dissipative force, the reactive component $\mathcal{F}_{\text{react}}$ is equal to zero in the case of running laser wave, since such a wave does not exhibit a Rabi frequency gradient $\nabla\mathcal{R}_0 = 0$.

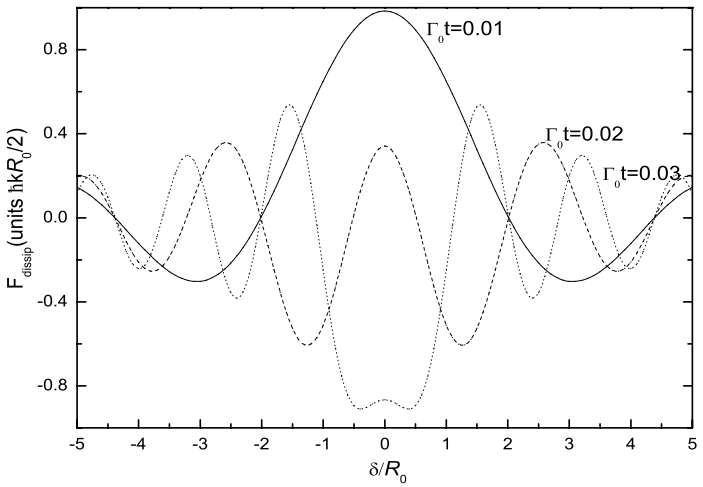

Fig. 4. The mean dissipative force as function of $\delta/\mathcal{R}_0$ with different values of $\Gamma_0 t$. The initial conditions and parameters are given in the text.

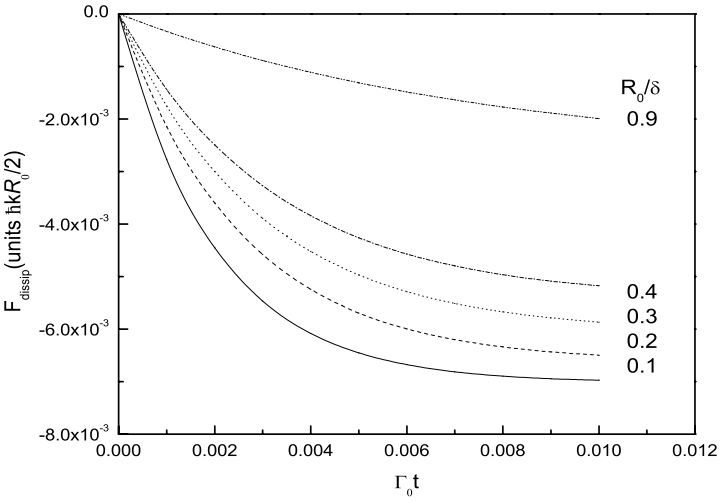

Fig. 5. The variation of the mean dissipative force $\Gamma_0 t$ with different values of detuning δ. See text for parameters and initial conditions.

5. SUMMARY

For an atom at rest or slowly moving, the mean radiative force is usually split into two parts (Gordon and Ashkin, 1980). The first part is related to the phase gradient of the laser wave (and to the quadrature part of the atomic dipole). The second part, is related to the intensity gradient of the laser (and to the in-phase atomic dipole). The first part, a dissipative force is well understood, and its various features, such as velocity dependence and momentum diffusion, have been analyzed in detail in terms of cycles involving absorption of laser photons and spontaneous emission of fluorescence photons (Wineland and Itano, 1979). On the other hand, the reactive force heats the atoms for a negative detuning and cools them for a positive one (Dalibard and Cohen-Tannoudji, 1985). In the majority of theories of atom-light interaction , the steady state approximation or other approximation were used to avoid the complexity of the solutions of the time evolution equations on the Bloch vector $\mathcal{B}(t)$ or on the internal density matrix $\rho(t)$. Recently (Bougouffa and Kamli, 2004; Kamli and Bougouffa, 2005), we have presented a separation approach based on a Riccati nonlinear differential equation appropriate for multi-level atoms interacting with single mode quantized field. In this work we generalized the separation approach of the coupled system. We derived a set of conditions that permitted the separation of the optical Bloch equations in the case of a pure radiative decay (Fig. 1). These conditions emerged in a natural manner from the requirements that the set of equations for the component $V(t)$ of Bloch vector are identical. The resulting equations are solved exactly

analytically and relevant quantities such as the dissipative and reactive forces are determined. The present approach persists with the new conditions of separation and without assuming the exact resonance, strong collisions or intense external field conditions (Carter *et al*., 2005). We emphasize here the fact that with the natural conditions, it is possible to obtain exact analytic solutions which reproduce the most important features of the mechanical effect of light on the moving atom. Although the numerical results reported here are obtained for the sodium atom and are valid for the initial conditions under consideration. We emphasize that the method proposed in this paper is valid for any initial conditions and is not restricted to the particular initial conditions that are considered here. This approach can be extended to the particular cases of translational and rotational aspects of atomic motion in laser light where the transient regime plays an important role and gives rise to new features in the subsequent dynamics.

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